1. Two fair 6-sided dice are thrown independently. Let X be the sum of the two numbers on the die faces, and let Y be the absolute value of the difference of the two numbers on the die faces. What is $\Pr(X > 2Y)$?

   (a) $\frac{24}{36}$

   (b) $\frac{26}{36}$

   (c) $\frac{22}{36}$

   (d) $\frac{28}{36}$

2. Let X and Y be two independent random variables. Then which of the following is/are true:

   (a) $E[XY] = 0$

   (b) $E[XY] = E[X]E[Y]$

   (c) If $X < 0$ then $Y > 0$

   (d) $E[e^X e^Y] = E[e^X]E[e^Y]$

3. Let A and B be two events in some probability space. Then which of the following is/are true:

   (a) If the intersection of A and B is empty then A and B are independent events.

   (b) If the intersection of A and B is empty then A and B are mutually exclusive.

   (c) If A is a subset of B, then $P(A) > P(B)$.

   (d) If A is a subset of B, then $P(A) \leq P(B)$.

4. Let F be the cumulative distribution function of a random variable X that takes values between 0 and 10. Then the following is/are true:

   (a) $F(2) > F(3)$

   (b) $F(2) \leq F(3)$

   (c) $F(11) = 1$

   (d) $F(-1) = 0$

5. Let X and Y be two independent random variables with variances 2 and 3, respectively. What is the variance of $4X - 3Y$?

   (a) 57

   (b) 59

   (c) 17

   (d) 5

6. Consider the following simple regression problem, with one dimensional input X and real valued target Y:

X	0.9	1.9	3.1	4.1	5.2	6.0
Y	11.0	9.1	7.3	5.1	3.2	1.1

Which of the following lines of the form $Y = mX + c$ has the least squared error (out of the options below)?

(a) $Y = 2X + 10$

(b) $Y = X + 8$

(c) $Y = -2X + 4$

(d) $Y = -2X + 10$

7. If $y = x^2$, then, $\frac{dx}{dy}$ is

(a) $2x$

(b) $2y$

(c) $\pm\frac{1}{2\sqrt{x}}$

(d) $\pm\frac{1}{2\sqrt{y}}$

8. The derivative of $\sin(5x)$ with respect to x is

(a) $\cos(5x)$

(b) $\cos x$

(c) $5\cos(5x)$

(d) $5\cos(x)$

9. Consider $f(x) = -5x^2 + 2x + 1$. Find the value of x for which $f(x)$ is maximum.

(a) $\frac{1}{5}$

(b) $-\frac{1}{5}$

(c) 5

(d) None of the above

10. What is the product of the eigenvalues of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$?

(a) 1

(b) 9

(c) 15

(d) Complex

11. In how many ways can you create a sample of 10 digits from $\{0, 1, 2, \ldots, 9\}$, if you sample with replacement?

(a) $10!$

(b) 10^{10}

(c) $\frac{10^{10}}{10!}$

(d) $100C_{10}$

12. The time to service a machine is an exponentially distributed random variable with parameter $\lambda = \frac{1}{2}$. If the service time exceeds 4 hours what is the probability that it exceeds 8 hours?

   (a) e^{-12}

   (b) e^{-2}

   (c) e^{-8}

   (d) e^{-4}

   (e) $e^{-4} + e^{-8}$

13. If a distribution is skewed to the right and has a median of 20, what will its mean be?

   (a) Equal to the mode

   (b) Less than 20

   (c) Equal to 20

   (d) Greater than 20

14. Which of the following is(are) not valid vectorspaces under addition and scalar multiplication, for finite n?

   (a) $\mathbb{R}^n$ - set of all n-dimensional real vectors.

   (b) $\mathbb{Z}^n$ - set of all n-dimensional integer vectors.

   (c) $\mathbb{P}^n$ - set of all n-th degree polynomials with real coefficients.

   (d) $\mathbb{C}^n$ - set of all n-dimensional complex-valued vectors.

15. Which of the following is(are) not a subspace?

   (a) A set containing only the zero vector.

   (b) Set of all 7 dimensional real vectors with its last entry to be zero.

   (c) Points in the plane, in $\mathbb{R}^3$, given by the equation $x + 2y = 3z$.

   (d) Set of all real-valued vectors in $\mathbb{C}^n$.

16. If $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are orthogonal vectors, then which of the following is(are) false?

   (a) They are linearly independent.

   (b) They span a subspace of dimension 3.

   (c) $(\mathbf{x}^T\mathbf{y})\mathbf{z}$ and $\mathbf{x}$ are linearly dependent.

(d) The rank of $\mathbf{z}\mathbf{x}^T$ is one.

17. Let $\mathbf{A}$ be a $2{\times}2$ matrix such that its column space is equal to its nullspace, then which of the following is(are) true?

    (a) Rank of $\mathbf{A}$ is 0.
    (b) Rank of $\mathbf{A}$ is 1.
    (c) Rank of $\mathbf{A}$ is 2.
    (d) Such a matrix cannot exist.

18. The determinant and trace of a 2×2 real symmetric matrix is given to be -1 and 0, respectively, then the eigenvalues of this matrix are

    (a) 1 and -1.
    (b) 1 and 0.
    (c) 0 and -1.
    (d) Insufficient information.

19. The eigenvalues of a 2×2 matrix are $1, -1$ and its singular values are $1, 0$. Then which of the following is(are) true?

    (a) Rank of the matrix is 2.
    (b) Rank of the matrix is 1.
    (c) Rank of the matrix is 0.
    (d) Such a matrix cannot exist.

20. Suppose the numbers 7, 5, 1, 8, 3, 6, 0, 9, 4, 2 are inserted in that order into an initially empty binary search tree. The binary search tree uses the usual ordering on natural numbers. What is the in-order traversal sequence of the resultant tree?

    (a) 7 5 1 0 3 2 4 6 8 9
    (b) 0 2 4 3 1 6 5 9 8 7
    (c) 0 1 2 3 4 5 6 7 8 9
    (d) 9 8 6 4 2 3 0 1 5 7

21. Which of the following is the correct recurrence for the worst case of Binary Search?

    (a) $T(n) = 2T(\frac{n}{2}) + O(1)$ and $T(1) = T(0) = O(1)$
    (b) $T(n) = T(n-1) + O(1)$ and $T(1) = T(0) = O(1)$
    (c) $T(n) = T(\frac{n}{2}) + O(1)$ and $T(1) = T(0) = O(1)$
    (d) $T(n) = T(n-2) + O(1)$ and $T(1) = T(0) = O(1)$

22. The number of edges in a fully connected directed graph with n vertices is

    (a) $n^2 + 1$
    (b) $n^2 - 1$
    (c) $n^2 - n$
    (d) $n^2 + n$

23. IIT Madras buses run from the hostel zone to the main gate starting from 6.00 am. Buses arrive at the main gate stop at the interval of 15 minutes. In other words, buses arrive at 6.00 am, 6.15 am, 6.30 am, ... If a staff arrives at the main gate at a time that is uniformly distributed between 6 am and 6.30 am, the probability that she waits less than 5 minutes for a bus is

    (a) $\frac{1}{2}$
    (b) $\frac{1}{3}$
    (c) $\frac{1}{4}$
    (d) $\frac{1}{5}$

24. The time complexity of Bubble sort algorithm is

    (a) $O(n)$
    (b) $O(\log n)$
    (c) $O(n^2)$
    (d) $O(n \log n)$

25. What will be the value of x after executing the following piece of code?

```
x = [1]
y = x
x += [2]
z = [3]
x = y + z
x *= 2
```

    Choose the correct answer:

    (a) $[1, 2, 3]$
    (b) $[1, 2, 1, 2, 3]$
    (c) $[1, 2, 1, 2, 1, 2, 3]$
    (d) $[1, 2, 1, 2, 3, 1, 2, 1, 2, 3]$

26. What will be the value of x after executing the following piece of code?

```
x = 19
x = x // 5
x = x // 2
x = x ~ 2
```

Choose the correct answer:

(a) 1

(b) 2

(c) 3

(d) 3.61

27. The Taylor series $f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ represents which one of the following functions?

(a) $x! + e^x$

(b) $\frac{e^x}{x!}$

(c) e^x

(d) $x!$

28. Find the equation of a line with a slope $m = 2$ and y-intercept $b = -3$.

(a) $y = -3x + 2$

(b) $y = 2x - 3$

(c) $y = \frac{2}{3}x + 1$

(d) $y = 2x + 3$

29. The function $f(x) = \sin(3x)$ is periodic with period

(a) $\frac{2\pi}{3}$

(b) π

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{3}$

30. If x, y and z are all positive and $x + y + z = 9$, the maximum value of xyz is

(a) 1

(b) $\sqrt{2}$

(c) 27

(d) 4

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