

New Course Proposal: IIT Kanpur

- **Course Title:** Arithmetic on quadratic fields
- **Department:** Department of Mathematics and Statistics
- **Course Number:** MTH3XX (UG level)
- **Proposing Instructor:** Somnath Jha
- **Other interested faculty members:**
- **Units:** 3-0-0-0 [9 credits]
- **Course Description:**

This is an introductory number theory course with a focus on quadratic fields. The main goal is to study the prime ideals and the units in the ring of integers of quadratic fields. In particular, we will explore the relation of the ideal class group of imaginary quadratic fields with the binary quadratic forms and also with the (Dedekind) Zeta function via the class number formula. We will also study quadratic reciprocity, and the Brahmagupta-Pell's equation. The course will be largely self contained and necessary tools from algebra and analysis will be developed along the way in the course.

- **Course content and lecture plan:**

- Abelian groups, subgroups and quotient groups, Lagrange's theorem, structure of finite abelian groups. [4 lectures]
- Commutative Rings, ideals, quotient rings, Prime ideals and maximal ideals, Polynomial rings, fields, field extensions. [4 lectures]
- the groups $\mathbb{Z}/n\mathbb{Z}$ and $(\mathbb{Z}/n\mathbb{Z})^*$, Euler's theorem and Wilson Theorem, Chinese Remainder Theorem [3 lectures]
- Frobenius map on finite fields, Classification of finite fields, the structure of the multiplicative subgroup of a finite field [3 lectures]
- Law of quadratic reciprocity, Quadratic fields, roots of unity, containment of quadratic fields inside cyclotomic fields [3 lectures]
- Unique Factorisation Domain (UFD), Principal ideal domain (PID), Euclidean Domain (ED), Ring of integers of quadratic fields, various examples. [4 lectures]
- Binary quadratic forms [3 lectures]
- ideal class groups of imaginary quadratic fields, relation with binary quadratic forms [4 lectures]
- Units in the ring of integers of real quadratic fields, Brahmagupta-Pell's equation, continued fractions [3 lectures]
- Riemann Zeta function, basic properties [3 lectures]
- class number formula for imaginary quadratic fields [5 lectures]

- Departments to which the proposed course will be of interest: MATHS, CSE, PHY.
- Prerequisites: MTH111M, MTH113M
- References:
 - A Classical Introduction to Modern Number Theory, **K. Ireland, M. Rosen**, Springer GTM.
 - An introduction to the Theory of Numbers, **I. Niven, H. S. Zuckerman, H. L. Montgomery**, Wiley.
 - Abstract Algebra, **D. S. Dummit, R. M. Foote**, Wiley.
 - Number Fields, **D. Marcus**, Universitext, Springer-Verlag.
 - Principles of mathematical analysis, **W. Rudin**, McGraw Hill.
 - Primes of the form $x^2 + ny^2$, 2nd ed., **D. A. Cox**, Pure and Applied Mathematics (Hoboken), John Wiley & Sons, Inc.
 - A course in Arithmetic, **J. P. Serre**, GTM 07, Springer-Verlag.
 - Algebraic Number Theory, **J. Neukirch**, Grundlehren der Mathematischen Wissenschaften 322, Springer-Verlag.
 - Problems in Algebraic Number Theory, **E. Jody and M. Ram Murty**, GTM 190, Springer-Verlag.
 - Complex Analysis, **E. Stein and R. Shakarchi**, Princeton University Press.

Dated: ____22 Sep 2025_____. Proposing Instructor: Somnath Jha

Dated: _____ DUGC Convener:_____

Dated:_____ Chairman, SUGC:_____

The course is approved / not approved