

Please answer **TWO** of the following three questions

**1. Algebra**

Consider a sequence  $\{a_n\}$ . The sum from the first term to the  $n$ th term of the sequence is denoted  $S_n$ , and satisfies:

$$S_n = 2a_n - n \times 2^{n+1} \quad (n = 1, 2, 3, \dots).$$

- a) Calculate  $a_1$  and  $a_2$ .
- b) Express  $a_{n+1}$  in terms of  $a_n$  and  $n$  by calculating  $S_{n+1} - S_n$ .
- c)  $b_n$  is defined as follows:

$$b_n = \frac{a_n}{2^n}.$$

Express  $b_n$ , then  $a_n$  in terms of  $n$ .

**2. Geometry**

In this question,  $x$  and  $y$  are real numbers.

- a) Sketch a graph of the following equation:

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}.$$

- b) Sketch the region that satisfies the following inequality:

$$x^2 + y^2 \leq |x| + |y|.$$

- c) Find the area of the region described in (b).

**3. Calculus**

Consider the function  $f(x)$ , defined as follows:

$$f(x) = \int_0^x (t - a)(t - b) dt,$$

where  $a$  and  $b$  are real constant values.

- a) Evaluate the function  $f(x)$  in terms of  $a$  and  $b$ .
- b) Find the values of  $a$  and  $b$  that satisfy the following three conditions on  $f(x)$ :
  - i.  $f(x)$  has a maximum or minimum at  $x = \frac{1}{2}$ ,
  - ii.  $f(a) - f(b) = \frac{1}{6}$ ,
  - iii.  $f'(0) > 0$ .
- c) Determine all maxima and minima of  $f(x)$ , and sketch the function.