Predicting orientations under manipulative actions

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Introduction

➢ What is the final orientation of an object when a robotic gripper does an action on it?

➢ How can we represent uncertainty over the space of rotations - $SO(3)$?

Consider a cube dropped by a gripper
Predict the face on which it falls - Classification
Predict the angular displacement - Regression

The Bingham Distribution

➢ Antipodally symmetric probability distribution over a hypersphere.

➢ Derived from a zero mean Gaussian on R^{d+1} constrained to lie on $S^d \subset R^{d+1}$

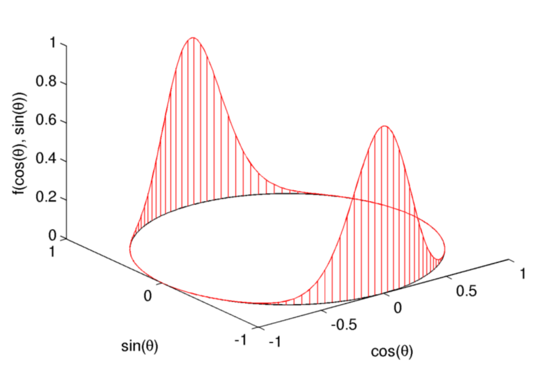
$$f(\mathbf{x};\Lambda, V) = \frac{1}{F(\Lambda)} exp(\mathbf{x}^T V \Lambda V^T \mathbf{x})$$

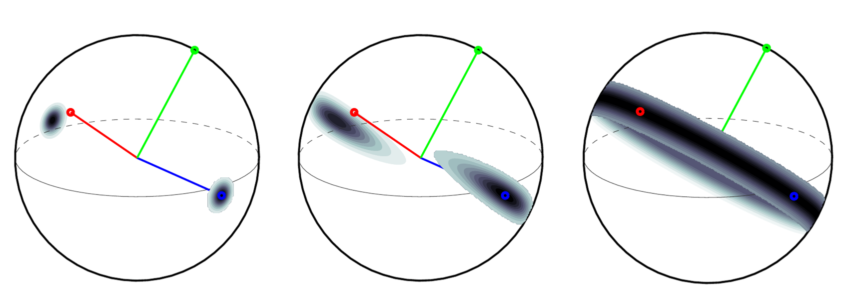
$$V = \begin{bmatrix} v_1 & v_2 & \dots & v_{d+1} \end{bmatrix}, v_i \in S^d$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \dots & \\ & & & \lambda_4 \end{bmatrix}$$

$$\lambda_1 \le \lambda_2 \le \dots \le \lambda_d \le 0$$

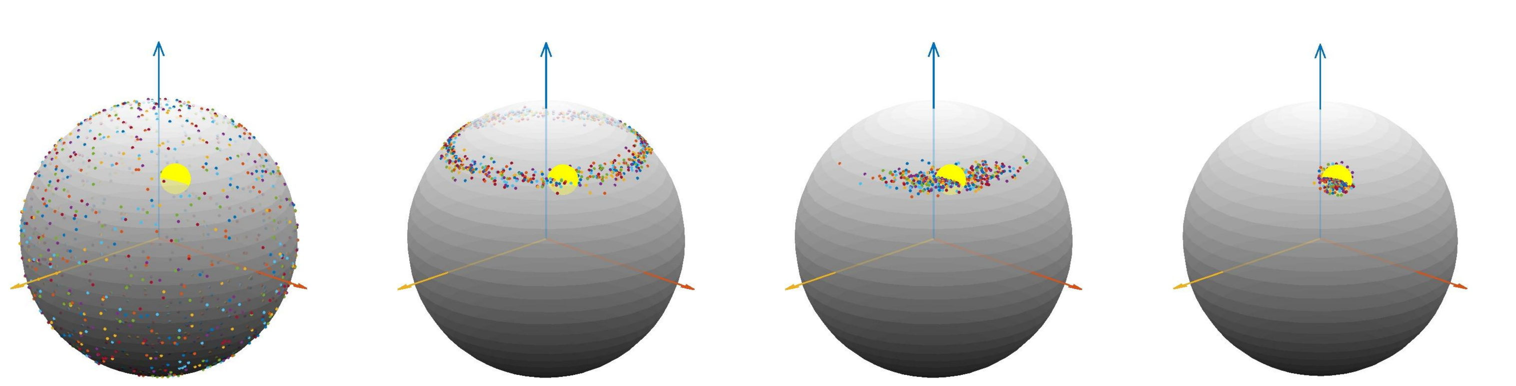
$F : normalization\ constant$





$(-100, -100)$ $(-100, -5)$ $(-100, 0)$

Bingham pdfs over S^1 and S^2 with varying concentration parameters



$(0, 0, 0)$ $(-900, -900, 0)$ $(-900, -900, -20)$ $(-900, -900, -900)$

For visualizing a quaternion Bingham distribution (over S^3), we can rotate a reference point (yellow) around the origin of a sphere by transformations corresponding to sampled quaternions.
The final position of the reference point is shown by the various points.
The concentration parameters are corresponding to eigen-quaternions, which are equivalent to zero rotation about X, Y and Z axes.

Approach

Gather Training Data
Drop cube in simulator with random orientations (sampled from a uniform Bingham distribution)

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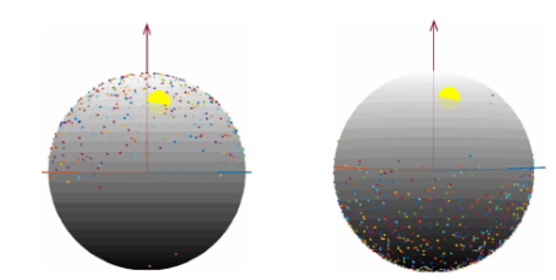
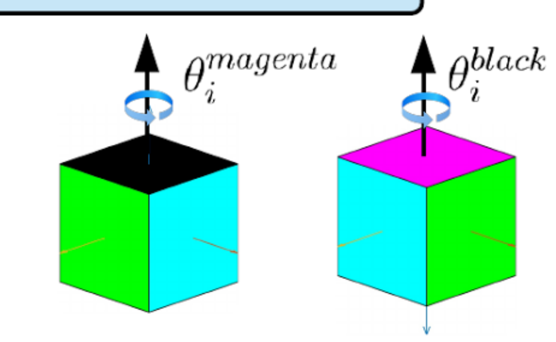
Label Data
(orientations falling on same face in one class)
Fit a Bingham Mixture Model to each class of initial orientations



All quaternions that "fell" on the "magenta" and the "black" face respectively.



The possible final orientations for the two classes of quaternions on the left.

↓

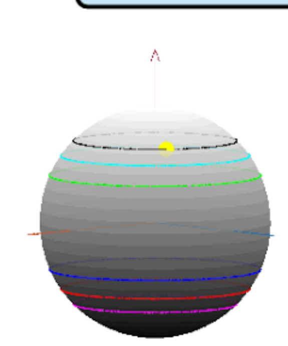
***MLE* Classification**

$$p(x|C_j) = \sum_{i=1}^{k} \alpha_i f(x; \Lambda_i, V_i), \quad x \in S^3$$

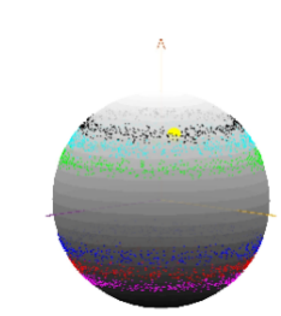
$$\hat{y} = \underset{j \in 1,\dots,J}{\text{argmax}}\ P(x|C_j)$$

$j = number\ of\ faces\ in\ the\ polyhedron, i.e\ no\ of\ classes$

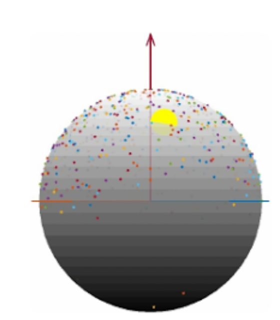
$k = number\ of\ components\ in\ the\ Bingham\ Mixture\ Model$



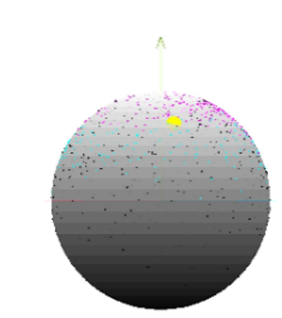
Each ring/color corresponds to various final orientations corresponding to each of the six classes, or the colored faces of a cube.



A BMM fitted to the data on the left. Each color corresponds to samples taken from the same mixture component.



Initial quaternions which "fell" on the same face, and hence belong to one class.



This BMM(fitted to the data on the left) has 3 components, corresponding to each color.

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Gaussian Process Regression from S^3 to S^1

$$Kernel : k(\mathbf{q}, \mathbf{q'}) = \sigma_f^2 exp\left(-\frac{(arccos^2(\langle \mathbf{q}, \mathbf{q'} \rangle))}{2\lambda^2}\right)$$

Train two independent GPs - one for sine and one for cosine of the angle (using GP Matlab Library)
Take tangent inverse to find predicted angle

$$E(\theta^*) = arctan(\theta^*_{sin}, \theta^*_{cos})$$

Reduced problem for parallel grippers

Parallel jaw gripper
Possible initial orientations (predictor) are in a "plane" (S^1) and so are the final ones (response)

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Reduce problem
To n circular regression problems (S^1 -> S^1) by extracting the "predictor" angle using the eigenvectors of the fitted Bingham distribution
n : no of faces of polyhedron

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Non-parametric methods
GP doesn't perform well. Use NPCirc library for circular regression (Nadaraya-Watson and local linear estimators)

Simulation Results

➢ Dropping with random orientations (GPML)

Classification accuracy = 90.53%

	1-nearest neighbour	SLERP (quaternion interpolation)	Gaussian Process
Mean	13.29˚	15.79˚	0.61˚
Std Dev	17.51˚	41.72˚	2.69˚
Max	132.99˚	179.98˚	7.22˚

➢ Parallel jaw gripper (NPCirc)

Angle of plane with horizontal	Face(0°)	Edge(45°)	Edge(30˚)
Classification	100%	56.72%	81.09%
Mean	2.2459e-04˚	1.21˚	0.29˚
Std Dev	0.0033˚	2.87˚	1.39˚
Max	0.0111˚	10.85˚	5.09˚

Future Work

➢ Regression in $SE(3)$.
➢ Improving classification accuracy.

References

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[2] M. Lang, O. Dunkley, and S. Hirche, "Gaussian process kernels for rotations and 6d rigid body motions," in IEEE International Conference on Robotics and Automation (ICRA), 2014
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