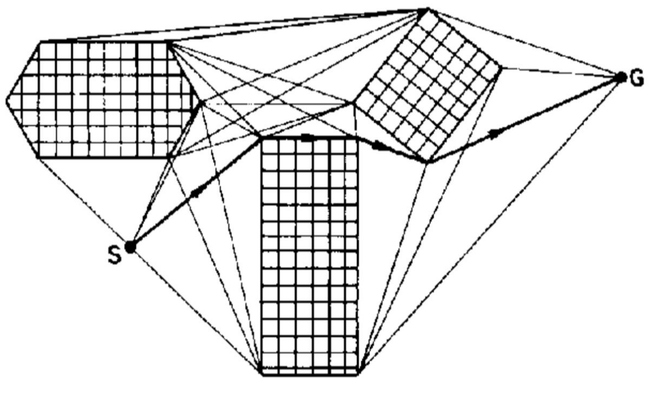
Finding the Shortest Path on a Sphere Surface Using Visibility Graphs

Saul Shanabrook
University of Massachusetts

Planning a military airstrike requires calculating many possible flight paths that navigate around threats. We needed a fast and accurate method for solving this subproblem.

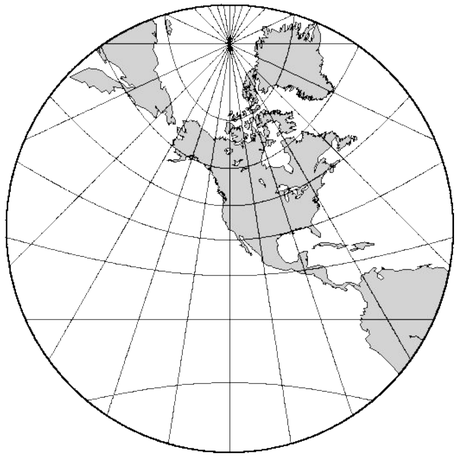
We present a method to find the shortest path between two points on the surface of a sphere that does not intersect a set of spherical polygons.

Solution



A **visibility graph** connects vertices of obstacles, source, and destination iff a straight line doesn't intersect any obstacle [1].

In the **Gnomonic Projection**, the shortest path between any two points on the sphere is projected as a straight line in the plane [2].

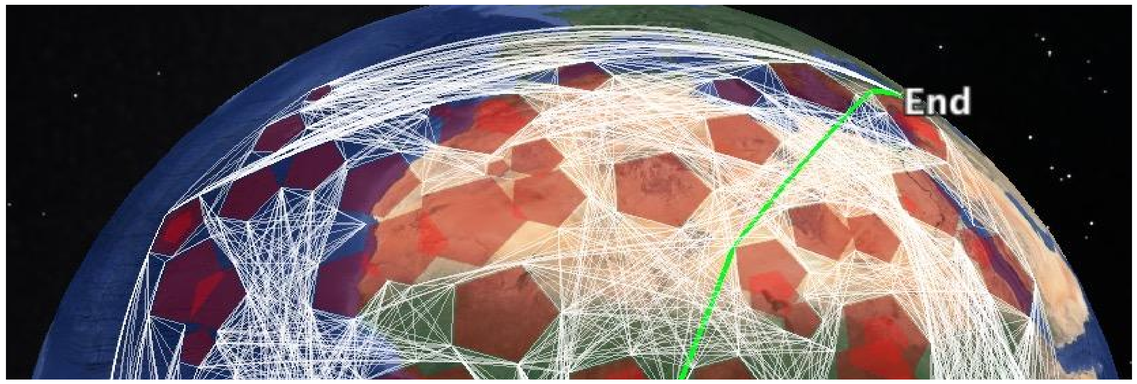


We combine these two pieces in **our method**:

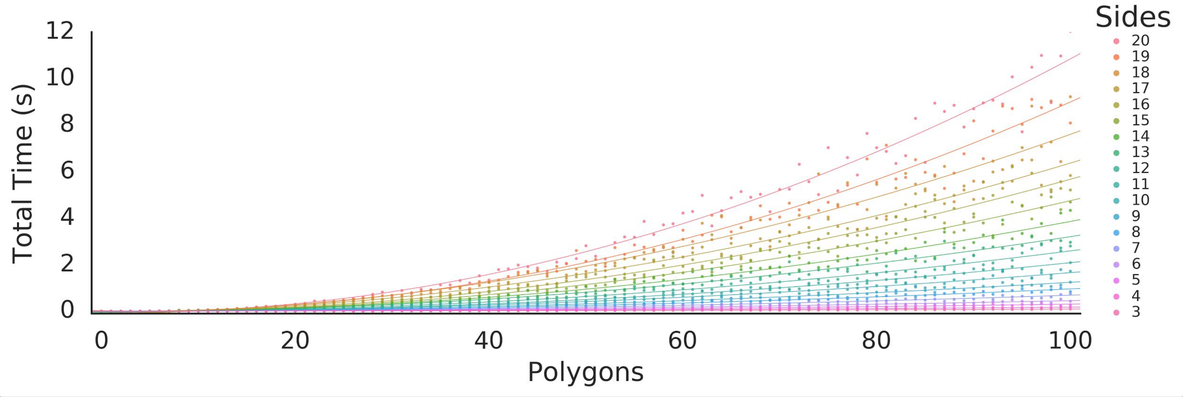
1. Map the points to a plane using the Gnomonic Projection.
2. Create a visibility graph from the points on the plane.
3. Search the visibility graph using A* with great circle distance[+] as the cost and heuristic.

[+] shortest direct path between two points on a sphere

Evaluation



We benchmarked random scenarios, varying the numbers of polygons (0 to 100) and sides per polygon (3 to 20).



Conclusions

- Runs relatively fast and can be further sped-up by choosing a faster algorithm to build the visibility graph.
- Average 20% runtime improvement over previous generative approach, which iteratively checks for obstacles and routes around them.
- Supports high resolution polygons.

[1] Lozano-Pérez, Tomás, and Michael A. Wesley. "An algorithm for planning collision-free paths among polyhedral obstacles." *Communications of the ACM* 22.10 (1979): 560-570.
[2] Weisstein, Eric W. "Gnomonic Projection." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/GnomonicProjection.html