A Minimal Jerk Trajectory Through Dynamic Support Polygon Reshaping for Modular Snake Robots

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Abstract

Snake robots are often presented as highly mobile systems where stable locomotion can be viewed as a form of manipulation with respect to the environment. This research addresses a related goal, to position a robotic end effector while modifying a statically stable base. In order to accomplish this task, a majority of the snake must be lifted while maintaining balance with respect to gravity. The proposed solution dynamically changes the *support polygon* to allow the end effector to maintain a minimal jerk trajectory in a cluttered environment. Experimental results empirically demonstrate that this approach presents a robust solution.

Modular Snake Robot

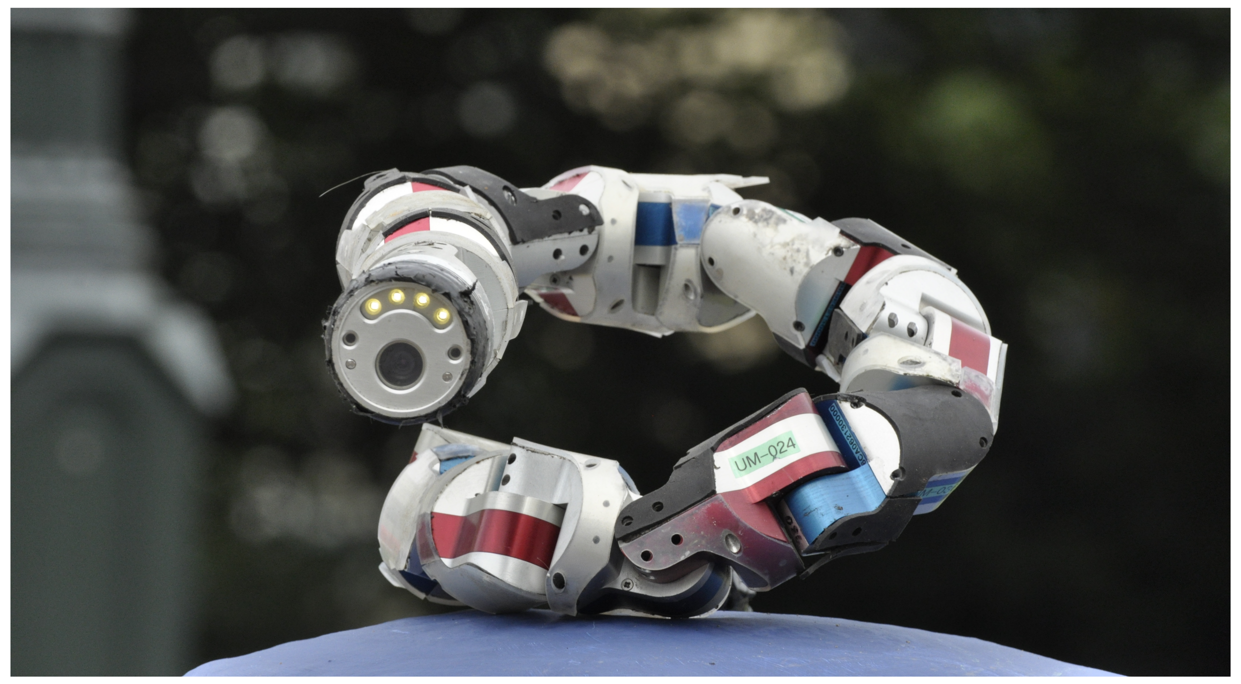

Figure 1: The Modular Snake Robot in action, developed by The Biorobotics Lab at CMU.

Jerk Trajectory Minimization

The model has two goals in a transition:

1. Raise the end-effector by two module lengths
2. **Minimize the jerk** during transition

Variable	Definition
$H(x)$	Cost Function
$X(t)$	Position Function
$\dddot{x}_i$	Jerk in Dimension i
z	End-Effector Position
a	Time Length of Transition

Table 2: Jerk Trajectory Variables

The second goal can be achieved by minimizing this **cost function**:

$$H(X(t)) = \frac{1}{2}\int_{t=0}^{a} \left(\sum_{i=0}^{n} \dddot{x}_i^{\,2}\right)$$

The solved differential equation:

$$z = z_i + (2\,l)\left[10\left(\frac{t}{a}\right)^3 - 15\left(\frac{t}{a}\right)^4 + 6\left(\frac{t}{a}\right)^5\right]$$

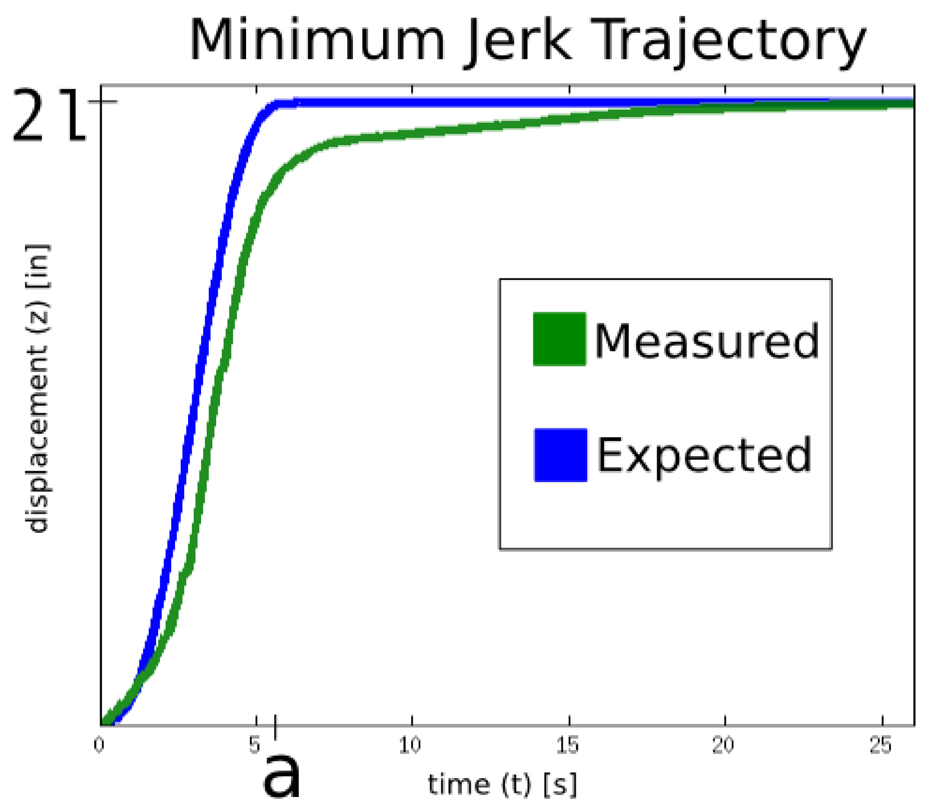

Figure 4: Minimum Jerk Trajectory for The End Effector

The **base shape** moves according to:

$$com_x = b\,sin\left(\frac{2\pi}{b}\right)$$
$$com_y = -b\,cos\left(\frac{2\pi}{b}\right)$$

For small values of b, **the jerk on the base is essentially zero**.

References

[1] C. Wright, A. Buchan, B. Brown, J. Geist, M. Schwerin, D. Rollinson, M. Tesch, and H. Choset, "Design and architecture of the unified modular snake robot," *2012 IEEE International Conference on Robotics and Automation*, pp. 4347–4354, May.

[2] N. Hogans, T. Flash, and N. Hogan, "The coordination of arm movements: an experimentally confirmed mathematical model," *The Journal of Neuroscience*, vol. 5, no. 7, pp. 1688–1703, 1985. [Online]. Available: http://www.jneurosci.org/content/5/7/1688.abstract

Support Polygon Reshaping

Variable	Definition
$R(\theta)$	The Polar Curve
b	The Distance Between Ends of Spiral
l	Length of a Module
θ	The Position on The Curve
r	The Approximated Radius
n	The Module Number
N	The Number of Modules
κ	Polar Curvature
ϕ_n	Joint Angles

Table 1: Support Polygon Variables

The approximate **curve**

$$R(\theta) = \frac{b}{2\pi}\left(\theta + \frac{r-b}{b}(2\pi)\right)$$

Discretization of the curve for each module:

$$R(n) = \frac{b\,n}{N} + l\,cot\left(\frac{2\pi}{N}\right) - b$$

For the snake, joint angles are needed [1]:

1. Calculate the **curvature**

$$\kappa = \frac{R^2 + 2\left(\frac{\delta R}{\delta n}\right)^2 - R\frac{\delta^2 R}{\delta n^2}}{\left(R^2 + \left(\frac{\delta R}{\delta n}\right)^2\right)^{\frac{3}{2}}}$$

2. **Integrate** over the modules

$$\phi_n = \int_{n-1}^{n+1} \kappa\,\delta n$$

With this, a stable supportive base with N modules and a leading module close to the center of the supporting polygon can be formed. This process works to form other shapes in two dimensions and can be extended with curvature in three dimensions.

Conclusions

This research has achieved:

1. Minimized jerk trajectory
2. Reshaped support polygon
3. Navigated snake into vertical pipe

Controller

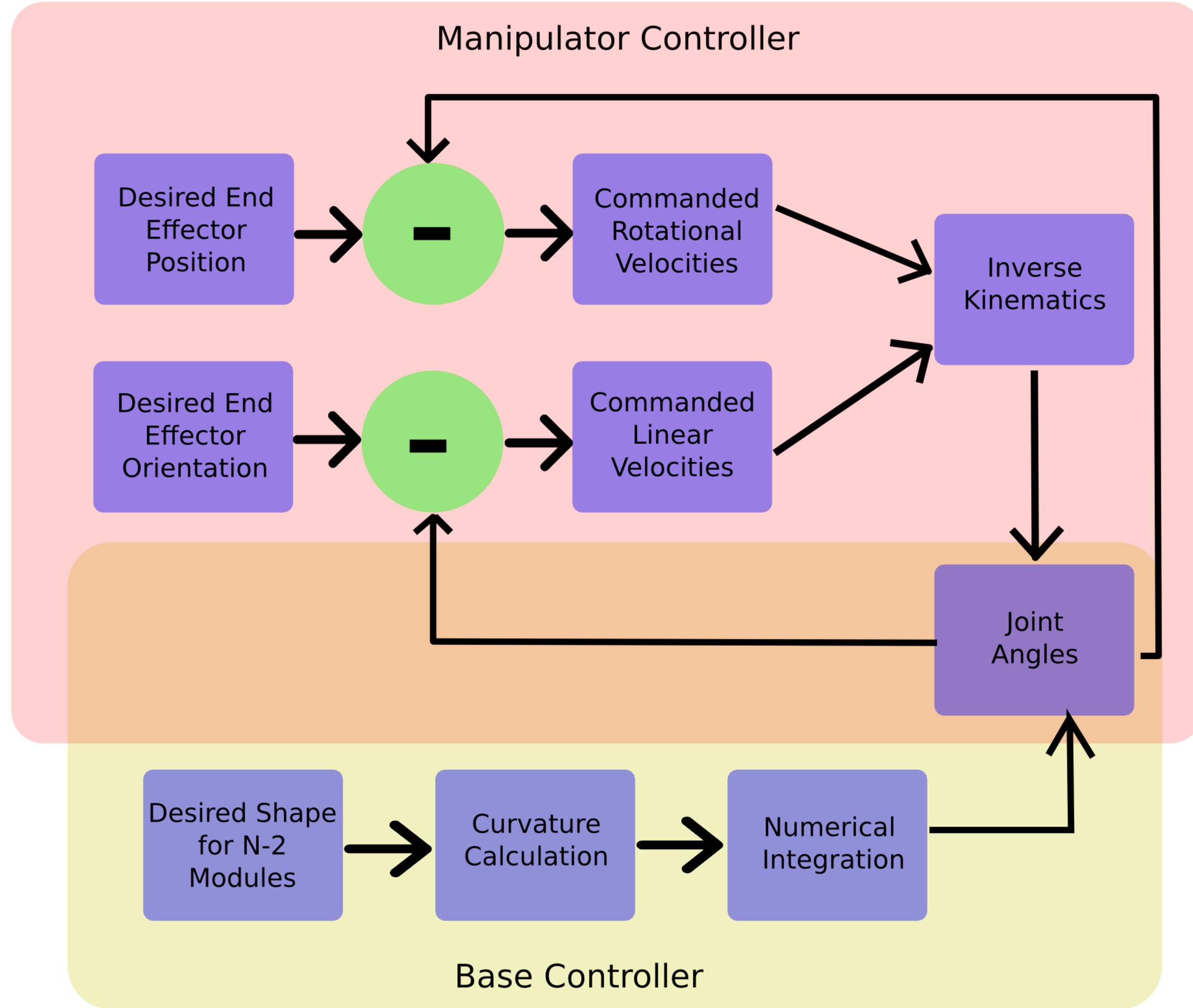

Figure 2: Outline of The Controller Used

Results

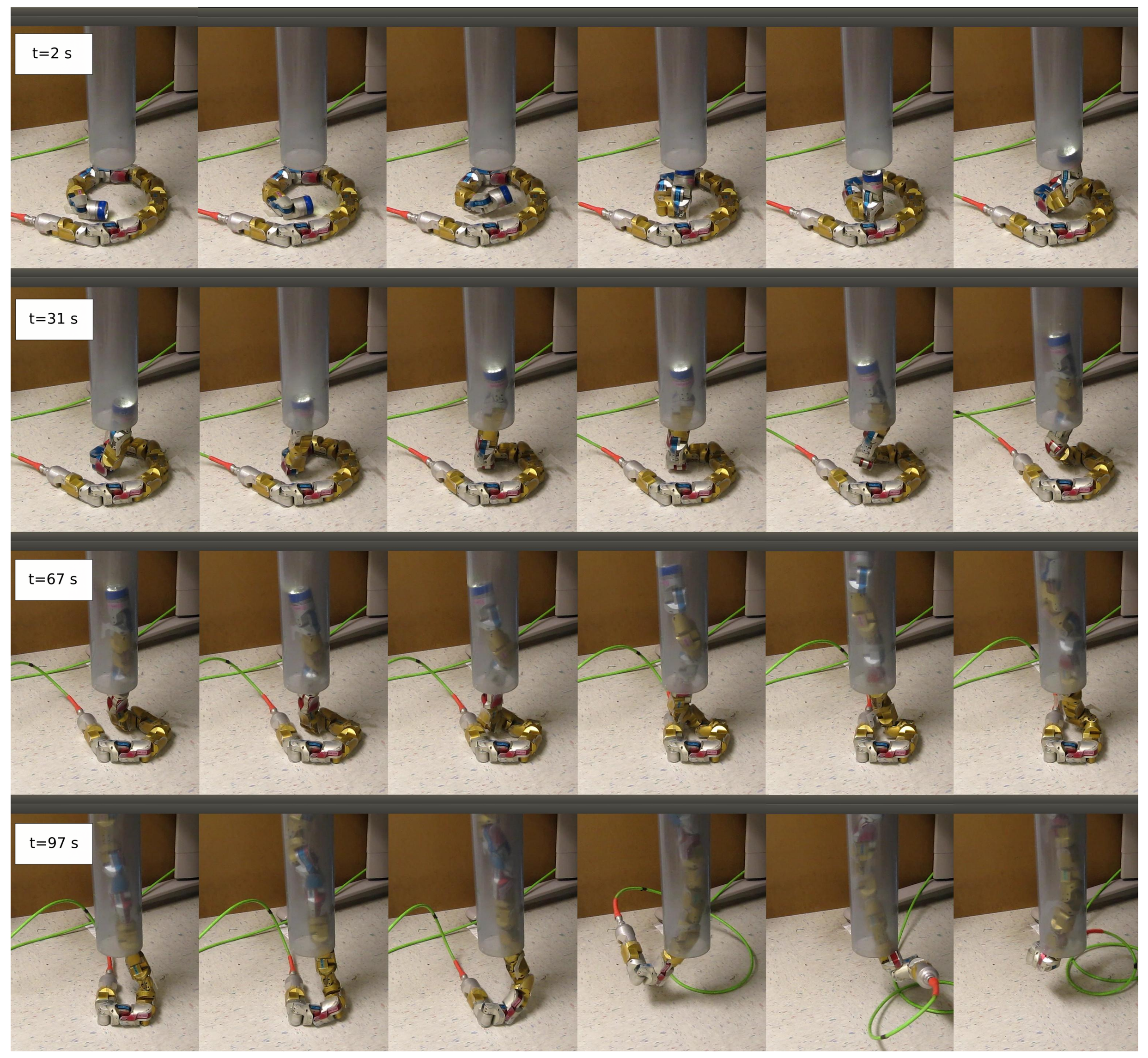

Figure 3: A Sequence Showing A Successful Experiment

Future Work

In order to advance this work, one could:

1. Consider rough or uneven terrain
2. Look into more autonomy
3. Conduct different end-effector behaviors