

Introduction to GAMS:
Take Home Midterm Exam

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Formulation of a General LP Problem

General Summation Notation:

$$\text{Max} \quad - \sum_M \sum_P c_{M,P} Y_{M,P} - \sum_P \sum_K \sum_N T_{P,K,N} S_{P,K,N}$$

$$\begin{aligned} \text{st.} \quad & - \sum_M Q_{N,M,P} Y_{M,P} + \sum_K S_{P,K,N} \leq 0, && \text{for all N,P} \\ & \sum_M u_{r,M,P} Y_{M,P} \leq b_{r,p}, && \text{for all r and p} \\ & \sum_P S_{P,K,N} \geq D_{K,N}, && \text{for all K, N} \\ & Y_{M,P}, S_{P,K,N} \geq 0, && \text{for all N, M, P, K} \end{aligned}$$

$c_{M,P}$: Production cost of utilizing process M in plant P;

$Y_{M,P}$: Amount of production process M utilized in plant P;

$T_{P,K,N}$: Transportation cost of shipping product N from plant P to customer K;

$S_{P,K,N}$: Quantity of product N shipped from plant P to customer K;

$Q_{N,M,P}$: Quantity yield of product N using process M in plant P;

$u_{r,M,P}$: Usage of resource r by process M in plant P.

$b_{r,p}$: Availability of resource r in plant P.

$D_{K,N}$: Fixed amount of demand (quantity needed) for product N by customer K.

GAMS Formulation

SET Products /X1, X2/
 Process /Y1, Y2/
 Plants /P1, P2/
 Customers /D1, D2/
 Resources /I1, I2/;

TABLE ProductionCost(Process,Plants)

		P1	P2
Y1	2	1	
Y2	1	2	;

PARAMETER Limit(Resources,plants)
 /I1.p1 100, I2.p1 200, I1.p2 200, I2.p2 100 /;

TABLE TransportCost(Plants, Customers, Products)

	D1.X1	D1.X2	D2.X1	D2.X2
P1	1	2	1	3
P2	2	1	3	1;

TABLE Yield(Products,Process,Plants)

	P1	P2
X1.Y1	4	3
X2.Y1	2	4
X1.Y2	2	4
X2.Y2	3	2;

TABLE ResourceUsage(Resources, Process, Plants)

	Y1.P1	Y1.P2	Y2.P1	Y2.P2
I1	1	1	1	1
I2	2	3	2	1;

TABLE Demand(Customers, Products)

	X1	X2
D1	22	21
D2	12	11;

VARIABLES

Profit;

POSITIVE VARIABLES

Production(Process, Plants)

Transport(Plants, Customers, Products);

EQUATIONS

TotalProfit

ProductionBalance(Products,Plants)

ResourceAvailability(Resources,Plants)

DemandLimit(Customers, Products) ;

TotalProfit..

Profit =E=

- SUM((Process,Plants),

ProductionCost(Process,Plants)

* Production(Process,Plants))

- SUM((Plants,Customers,Products),

TransportCost(Plants, Customers, Products)

* Transport(Plants, Customers, Products)) ;

ProductionBalance(Products,Plants)..

- SUM(Process,

Yield(Products, Process, Plants)

* Production(Process, Plants))

+ SUM(Customers,

Transport(Plants,Customers,Products))

=L= 0 ;

ResourceAvailability(Resources,plants)..

SUM(Process,

ResourceUsage(Resources,Process,Plants)

* Production(Process,Plants))

=L= Limit(Resources,Plants);

DemandLimit(Customers,Products)..

SUM(Plants,Transport(Plants,Customers,Products))

=G= Demand(Customers,Products) ;

MODEL TakeHome /ALL/;

SOLVE TakeHome USING LP MAXIMIZATION Profit;

STEPS to Transform LP Formulation to GAMS Formulation

1. SET definitions:

Define a subscript for each physical phenomena set which has multiple members and use different subscripts when considering subsets of the subscript set or different physical phenomena.

2. Data entry using parameter, table, or scalar GAMS statement

Define a unique symbol with a subscript for each manipulatable item.

3. Variables specification

Make sure that variable has the same subscript in each place it occurs

4. Equations specification – declaration and algebraic structure

Be careful to identify the conditions under which each equation exists and use subscripts to identify those conditions.

5. Model statement

6. Solve statement

STEPS to Transform LP Formulation to GAMS Formulation

1. SET definitions



```

SET Products /X1, X2/
    Process /Y1, Y2/
    Plants /P1, P2/
    Customers /D1, D2/
    Resources /I1, I2/;
    
```

2. Data entry using



a. parameter

b. table

$c_{M,P}$: Production cost

$b_{r,P}$: Availability of resource

$T_{P,K,N}$: Transportation cost

$Q_{N,M,P}$: Quantity yield of product

$u_{r,M,P}$: Usage of resource

$D_{K,N}$: Fixed amount of demand

```

TABLE ProductionCost(Process,Plants)
    
```

	P1	P2
Y1	2	1
Y2	1	2

```

PARAMETER Limit(Resources,plants)
    
```

```

/I1.p1 100, I2.p1 200, I1.p2 200, I2.p2 100 /;
    
```

```

TABLE TransportCost(Plants, Customers, Products)
    
```

	D1.X1	D1.X2	D2.X1	D2.X2
P1	1	2	1	3
P2	2	1	3	1

```

TABLE Yield(Products,Process,Plants)
    
```

	P1	P2
X1.Y1	4	3
X2.Y1	2	4
X1.Y2	2	4
X2.Y2	3	2

```

TABLE ResourceUsage(Resources, Process, Plants)
    
```

	Y1.P1	Y1.P2	Y2.P1	Y2.P2
I1	1	1	1	1
I2	2	3	2	1

```

TABLE Demand(Customers, Products)
    
```

	X1	X2
D1	22	21
D2	12	11

3. Variable specifications



VARIABLES
 Profit;
POSITIVE VARIABLES
 Production(Process, Plants)
 Transport(Plants, Customers, Products);

4. Equation specifications

a. declaration



EQUATIONS
 TotalProfit
 ProductionBalance(Products,Plants)
 ResourceAvailability(Resources,Plants)
 DemandLimit(Customers, Products) ;

b. algebraic structure

General Summation Notation:

$$\begin{aligned}
 \text{Max} \quad & - \sum_M \sum_P c_{M,P} Y_{M,P} - \sum_P \sum_K \sum_N T_{P,K,N} S_{P,K,N} \\
 \text{st.} \quad & - \sum_M Q_{N,M,P} Y_{M,P} + \sum_K S_{P,K,N} \leq 0, \quad \text{for all } N,P \\
 & \sum_M U_{M,P} Y_{M,P} \leq b_{N,P}, \quad \text{for all } r \text{ and } p \\
 & \sum_P S_{P,K,N} \geq I_{K,N}, \quad \text{for all } K, N \\
 & Y_{M,P}, S_{P,K,N} \geq 0, \quad \text{for all } N, M, P, K.
 \end{aligned}$$

TotalProfit..
 Profit =E=
 - SUM((Process,Plants),
 ProductionCost(Process,Plants)
 * Production(Process,Plants))
 - SUM((Plants,Customers,Products),
 TransportCost(Plants, Customers, Products)
 * Transport(Plants, Customers, Products)) ;
ProductionBalance(Products,Plants)..
 - SUM(Process,
 Yield(Products, Process, Plants)
 * Production(Process, Plants))
 + SUM(Customers,
 Transport(Plants,Customers,Products))
 =L= 0 ;
ResourceAvailability(Resources,plants)..
 SUM(Process,
 ResourceUsage(Resources,Process,Plants)
 * Production(Process,Plants))
 =L= Limit(Resources,Plants);
DemandLimit(Customers,Products)..
 SUM(Plants,Transport(Plants,Customers,Products))
 =G= Demand(Customers,Products) ;

5. Model statement



MODEL TakeHome /ALL/;
SOLVE TakeHome **USING LP MAXIMIZATION** Profit;

6. Solve statement

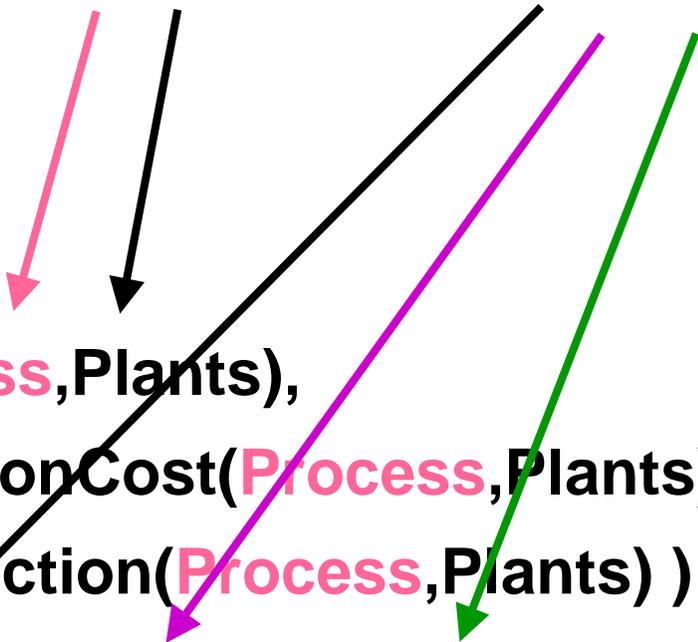
Let look at **Equations Specification** in details.

$$-\sum_M \sum_P c_{M,P} Y_{M,P} - \sum_P \sum_K \sum_N T_{P,K,N} S_{P,K,N}$$

TotalProfit..

Profit =E=

- **SUM**((Process,Plants),
 ProductionCost(Process,Plants)
 * Production(Process,Plants))
 - **SUM**((Plants,Customers,Products),
 TransportCost(Plants, Customers, Products)
 * Transport(Plants, Customers, Products)) ;



$$-\sum_M Q_{N,M,P} Y_{M,P} + \sum_K S_{P,K,N} \leq 0 \quad \text{for all } N, P$$

Production Balance(Products,Plants)..

- SUM(Process,

Yield(Products, Process, Plants)

* Production(Process, Plants))

+ SUM(Customers,

Transport(Plants, Customers, Products))

=L= 0 ;

$$\sum_M u_{r,M,P} Y_{M,P} \leq b_{r,P} \quad \text{for all } r, P$$

ResourceAvailability(**Resources**,Plants)..

SUM(**Process**,

ResourceUsage(**Resources**,**Process**,Plants)

* Production(**Process**,Plants))

=L=

Limit(**Resources**,Plants);

$$\sum_P S_{P,K,N} \geq D_{K,N} \quad \text{for all } K, N$$

DemandLimit(**Customers,Products**)..

SUM(Plants,

Transport(Plants,**Customers,Products**))

=G=

Demand(**Customers,Products**) ;