Introduction to Computable General Equilibrium Model (CGE)

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Course Outline

- **Overview of CGE**

- **An Introduction to the Structure of CGE**

- **An Introduction to GAMS**

- **Casting CGE models into GAMS**

- **Data for CGE Models & Calibration**

- **Incorporating a trade & a basic CGE application**

- Evaluating impacts of policy changes and casting nested functions & a trade in GAMS

- Mixed Complementary Problems (MCP)

This Week's Road Map

- Incorporating a simple trade relationship in a CGE model

- SAM in GAMS

- Calibration in GAMS

- Basic CGE application (Shoven & Whalley)

Incorporating Trade – modifications to a basic CGE model

- **Basic relationships for a simple CGE model**

  1. Supply–Demand
  2. Zero profit condition
  3. Factor demand by producers
  4. Product demand by households
  5. Income balance constraint for households
  6. Government balance

  7. **Trade Balance**

     assumption:

     - Small open-economy
     - Only goods are tradable.
     - Goods are homogenous across countries

     (ignore the CET and ARMINGTON approaches for now)

Incorporating Trade – modifications to a basic CGE model

1. The commodity market balance $\qquad 0 \le P_j \qquad \perp$

$$\sum_h X_{jh} + \sum_{j1} a_{j,j1} Q_{j1} - Q_j + s_j R / P_j$$

$$\boxed{+ QExp_j - QImp_j} \qquad \le 0 \qquad \forall\, j$$

2. The factor market balance

   => no change (assuming no trade on factors)

3. The profit condition

   => no change

4. The household income equation

   => no change

Incorporating Trade – modifications to a basic CGE model

5. The government tax revenue balance $0 \le R \qquad \perp$

$$R \le \sum_j t_j P_j Q_j - \sum_h t_h (W_L \overline{L}_h + W_K \overline{K}_h - F_h)$$

$$+ \sum_j \left(t_{lj} W_L L_j + t_{kj} W_K K_j \right)$$

$$+ \sum_{j \in ex} tExp_j QExp_j P_j + \sum_{j \in im} tImp_j QImp_j P_j$$

Incorporating Trade – modifications to a basic CGE model

6. The domestic (US) trade price equations

   Exports:

   $$0 \leq PWExp_j \quad \perp$$

   $$PExp_j \leq (PWExp_j \times Exchange) - TransCost \quad \forall \ j \in ex$$

   Imports:

   $$0 \leq PWImp_j \quad \perp$$

   $$PImp_j \leq (PWImp_j \times Exchange) + TransCost \quad \forall \ j \in im$$

Incorporating Trade – modifications to a basic CGE model

7. The domestic (US) trade quantity equations

Exports:

$$0 \le QExp_j \quad \perp$$

$$QExp_j \le f(PWExp_j, \varepsilon) \qquad \forall \ j \in ex$$

where ε and μ are price elasticities

$-INF < \varepsilon < 0$ and $0 < \mu < INF$

Imports:

$$0 \le QImp_j \quad \perp$$

$$QImp_j \le f(PWImp_j, \mu) \qquad \forall \ j \in im$$

Note: Shoven and Whalley showed that $\varepsilon \le -1$ so that the export-demand curve slopes downward and the import-supply slopes upward.

Incorporating Trade – modifications to a basic CGE model

8. The domestic & trade price relationship

Export tax => price received by domestic producers from exports NOT = from domestic goods

Import tax => price paid by domestic consumers for imports NOT = for domestic goods

Exports: $0 \leq PExp_j \quad \perp \quad PExp_j = (1 - tExp_j)P_j \quad \forall\, j \in ex$

Imports: $0 \leq PImp_j \quad \perp \quad PImp_j = (1 + tImp_j)P_j \quad \forall\, j \in im$

9. The zero trade balance

: The exchange rate is used to clear a market where values of imports are equal to values of exports.

$$0 \leq Exchange \quad \perp \quad \sum_{j \in im} PWImp_j QImp_j \quad \leq \quad \sum_{j \in ex} PWExp_j QExp_j$$

SAM

Recall: SAM Table used in the class example last week

	Food Act	NonFood Act	Food	NonFood	Labor	Capital	Household	Government	Tax	ROW	Total
Food Activity	0	0	1430	0				0			1430
NonFood Activity	0	0	0	850				0			850
Food	0	100	0	0			1020	10		300	1430
NonFood	200	0	0	0			910	40		0	1150
Labor	900	200									1100
Capital	300	500									800
Household					1100	800			30		1930
Government			0				0		50		50
Tax	30	50							0		80
ROW			0	300							300
Total	1430	850	1430	1150	1100	800	1930	50	80	300	

SAM in GAMS

STEPS entering SAM Table in GAMS

1. Define **SETS** and elements of **SETS**

2. Create SUBSETS from SETS

3. Enter data using a **TABLE** statement

SAM in GAMS

STEP 1: Define **SETS** and elements of **SETS**

```
SET
 AllSets           including all of items used in the model
 /FoodAct          Food Activity
  NonFoodAct       Non Food Activity
  Food             Food sector
  NonFood          Non food sector
  Labor            Labor
  Capital          Capital
  HouseH           Household
  Govern           Government
  Tax              Tax
  ROW              ROW
  Total            Total
 /;

ALIAS(AllSets,AllSets1);
```

SAM in GAMS

STEP 2: Create SUBSETS from SETS

```
SET
 Activity(AllSets)  Using subset of ALLSET for activity
 /FoodAct           Food Activity
  NonFoodAct        Non Food Activity
 /


 Sector(AllSets)    Using subset of ALLSET for sector
 /Food              Food sector
  NonFood           Non food sector
 /


 Factor(AllSets)    Using subset of ALLSET for factor
 /Labor             Labor
  Capital           Capital
 /


 Consumer(AllSets)  Using subset of ALLSET for final demand consumers
 /HouseH            Household
  Govern            Government
  ROW               ROW
 /  ;
```

SAM in GAMS

STEP 3: . Enter data using a **TABLE** statement

TABLE   SAMValue(AllSets,AllSets1)   Reading in SAM original table in dollar values

	FoodAct	NonFoodAct	Food	NonFood	Labor	Capital	Househ	Govern	Tax	ROW	Total
FoodAct			1430								1430
NonFoodAct				850							850
Food		100					1020	10		300	1430
NonFood	200						910	40			1150
Labor	900	200									1100
Capital	300	500									800
HouseH					1100	800			30		1930
Govern									50		50
Tax	30	50									80
ROW				300							300
Total	1430	850	1430	1150	1100	800	1930	50	80	300	

;

Calibration in GAMS

Using GAMS to check if **SAM** accounts are balanced before starting a calibration

=> a column sum   =   a row sum

```
PARAMETER
 CheckSam(*,*)     Checking if column totals equal row totals  ;

 CheckSam("total-column",AllSet1)
   = sum(AllSet,SAMValue(AllSet,AllSet1))  ;
 CheckSam("total-row",AllSet)
   = sum(AllSet1,SAMValue(AllSet,AllSet1))  ;

OPTION   CheckSam:0:1:1;
DISPLAY CheckSam;
```

Calibration in GAMS

Output from GAMS

```
----       93 PARAMETER CheckSam  Checking if column totals equal row totals

                    FoodAct    NonFoodAct        Food      NonFood        Labor

total-column           1430           850        1430         1150         1100
total-row              1430           850        1430         1150         1100

            +        Capital        HouseH      Govern          Tax          ROW

total-column            800          1930          50           80          300
total-row               800          1930          50           80          300
```

Calibration in GAMS

Creating a parameter to flag on accounts that are not balanced

```
SET
 BadCheckSAM(AllSets)  Show me which account has a problem ;
 BadCheckSAM(AllSet)
   = YES$ ( CheckSam("total-column",AllSet)
              NE CheckSam("total-row",AllSet)) ;
DISPLAY BadCheckSAM;


----     97 SET      BadCheckSAM  Show me which account has a problem

FoodAct,    Tax


----     99 PARAMETER CheckSam  Checking if column totals equal row totals

                FoodAct  NonFoodAct       Food    NonFood      Labor
total-column       1530         850       1430       1150       1100
total-row          1430         850       1430       1150       1100

                 +  Capital      HouseH     Govern        Tax        ROW
total-column          800        1930         50         80        300
total-row             800        1930         50        180        300
```

Calibration in GAMS

STEPS used in a calibration:

(1). Define parameters used in a calibration

```
PARAMETER
  Alpha(AllSets,AllSets1)        calibrate alpha share parameters
  BigA(AllSets)                  calibrate technical change parameters
  ProductQ(AllSets)              replicate production
  FactorQ(AllSets)               replicate factor demand
  Price(AllSets)                 prices
  SAM(AllSets,AllSets1)          SAM Table in after normalizing prices
     ;
```

Calibration in GAMS

(2). Decide on units for goods and factors so that prices and quantities are separately obtained.

Normalizing prices to unity

```
Price(Factor)   = 1;
Price(Sector)   = 1;
Price(Activity) = 1;
```

Modifying SAM TABLE so that units are in quantities

```
SAM(Factor,Activity) = SAMValue(Factor,Activity)/Price(Factor);
SAM(Activity,Sector) = SAMValue(Activity,Sector)/Price(Sector);
SAM(Sector,Consumer) = SAMValue(Sector,Consumer)/Price(Sector);
SAM(Consumer,Factor) = SAMValue(Consumer,Factor)/Price(Factor);
```

Calibration in GAMS

Recall: **The numerical example for the Cobb Douglas production function used last week**

$$Q_F = A_F K_F^{\alpha} L_F^{1-\alpha}$$

$$\alpha_F = \frac{r_F K_F}{P_F Q_F} = \frac{1 \times 300}{1 \times 1200} = 0.25$$

```
Alpha(Factor,Activity)
 = ( Price(Factor)*SAM(Factor,Activity))
     /( Price(Activity)*SUM(AllSet,SAM(AllSet,Activity)) ) ;
OPTION  Alpha:3:1:1;DISPLAY Alpha;

----    137 PARAMETER Alpha   calibrate alpha share parameters


            FoodAct   NonFoodAct


Labor         0.750       0.286
Capital       0.250       0.714
```

$$A_F = \frac{Q_F}{K_F^\alpha L_F^{1-\alpha}} = \frac{1200}{300^{0.25} 900^{0.75}} = 1.75$$

```
BigA(Activity)
 = SUM(AllSet,SAM(AllSet,Activity))
    /(PROD(Factor,SAM(Factor,Activity)**Alpha(Factor,Activity) ) ) ;
OPTION  BigA:3:0:1;DISPLAY BigA;
```

```
----      144 PARAMETER BigA  calibrate technical change parameters


FoodAct    1.755
NonFoodAct 1.819
```

Calibration in GAMS

Replication check after a calibration

$$Q_F = A_F K_F^{\alpha} L_F^{1-\alpha}$$
$$= 1.75 \times 300^{0.25} \times 900^{0.75} = 1200$$

```
ProductQ(Activity)
 = BigA(Activity)
    *(PROD(Factor,SAM(Factor,Activity)**Alpha(Factor,Activity) ) );
OPTION  ProductQ:3:0:1;DISPLAY ProductQ;
```

```
----    150 PARAMETER ProductQ  replicate production

FoodAct      1200.000
NonFoodAct    700.000
```

Calibration in GAMS

$$K_F = \frac{Q_F}{A_F}\left(\frac{\alpha w}{(1-\alpha)r}\right)^{1-\alpha}$$

$$= \frac{1200}{1.75}\left(\frac{0.25 \times 1}{0.75 \times 1}\right)^{0.75} = 300$$

```
FactorQ(Activity)
  = (ProductQ(Activity)/BigA(Activity))
      *(   (Alpha("Capital",Activity)*Price("Labor"))
            /(Alpha("Labor",Activity)*Price("Capital"))
        )**(1-Alpha("Capital",Activity))      ;

 OPTION  FactorQ:3:0:1;DISPLAY FactorQ;

----    160 PARAMETER FactorQ  replicate factor demand


FoodAct    [300.000]
NonFoodAct 500.000
```

Wrap Up

- Incorporating the trade relationship in the CGE model

- SAM in GAMS

- Calibration in GAMS

Next:

- MCP

- Evaluating results from the CGE model

- Incorporating nested functions and a trade in GAMS

- Manne A. S. and R. G. Richels. "An alternative approach to establishing trade-offs among greenhouse gases." *Nature* 410, 675-677 (2001).

Reference:

Shoven, J. B. and J. Whalley. "Applying general equilibrium." *Surveys of Economic Literature*, Chapter 7, 1998.